NAG Toolbox for MATLAB

f08na

1 Purpose

f08na computes the eigenvalues and, optionally, the left and/or right eigenvectors for an n by n real nonsymmetric matrix A.

2 Syntax

```
[a, wr, wi, vl, vr, info] = f08na(jobvl, jobvr, a, 'n', n)
```

3 Description

The right eigenvector v_j of A satisfies

$$Av_j = \lambda_j v_j$$

where λ_j is the jth eigenvalue of A. The left eigenvector u_j of A satisfies

$$u_j^{\mathrm{H}} A = \lambda_j u_j^{\mathrm{H}}$$

where u_j^{H} denotes the conjugate transpose of u_j.

The matrix A is first reduced to upper Hessenberg form by means of orthogonal similarity transformations, and the QR algorithm is then used to further reduce the matrix to upper quasi-triangular Schur form, T, with 1 by 1 and 2 by 2 blocks on the main diagonal. The eigenvalues are computed from T, the 2 by 2 blocks corresponding to complex conjugate pairs and, optionally, the eigenvectors of T are computed and backtransformed to the eigenvectors of A.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobvl – string**

If **jobvl** = 'N', the left eigenvectors of A are not computed.

If **jobvl** = 'V', the left eigenvectors of A are computed.

Constraint: **jobvl** = 'N' or 'V'.

2: **jobvr – string**

If **jobvr** = 'N', the right eigenvectors of **a** are not computed.

If **jobvr** = 'V', the right eigenvectors of **a** are computed.

Constraint: **jobvr** = 'N' or 'V'.

3: **a**(**lda**,∗) – **double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n matrix A.

5.2 Optional Input Parameters

1: **n** – **int32 scalar**

Default: The first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldvl, ldvr, work, lwork

5.4 Output Parameters

1: **a**(**lda**,∗) – **double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

a has been overwritten.

2: **wr**(∗) – **double array**

3: **wi**(∗) – **double array**

Note: the dimension of the arrays **wr** and **wi** must be at least $\max(1, \mathbf{n})$.

wr and **wi** contain the real and imaginary parts, respectively, of the computed eigenvalues. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having the positive imaginary part first.

4: **vl**(**ldvl**,∗) – **double array**

The first dimension, **ldvl**, of the array **vl** must satisfy

if **jobvl** = 'V', **ldvl** $\geq \max(1, \mathbf{n})$;
ldvl ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$ if **jobvl** = 'V', and at least 1 otherwise

If **jobvl** = 'V', the left eigenvectors u_j are stored one after another in the columns of **vl**, in the same order as their corresponding eigenvalues.

If **jobvl** = 'N', **vl** is not referenced.

If the jth eigenvalue is real, then $u_j = \mathbf{vl}(:, j)$, the jth column of **vl**.

If the jth and $(j+1)$st eigenvalues form a complex conjugate pair, then $u_j = \mathbf{vl}(:, j) + i \times \mathbf{vl}(:, j+1)$ and $u_{j+1} = \mathbf{vl}(:, j) - i \times \mathbf{vl}(:, j+1)$.

5: **vr**(**ldvr**,∗) – **double array**

The first dimension, **ldvr**, of the array **vr** must satisfy

if **jobvr** = 'V', **ldvr** $\geq \max(1, \mathbf{n})$;
ldvr ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1,\mathbf{n})$ if **jobvr** = 'V', and at least 1 otherwise

If **jobvr** = 'V', the right eigenvectors v_j are stored one after another in the columns of **vr**, in the same order as their corresponding eigenvalues.

If **jobvr** = 'N', **vr** is not referenced.

If the jth eigenvalue is real, then $v_j = \mathbf{vr}(:,j)$, the jth column of **vr**.

If the jth and $(j+1)$st eigenvalues form a complex conjugate pair, then $v_j = \mathbf{vr}(:,j) + i \times \mathbf{vr}(:,j+1)$ and $v_{j+1} = \mathbf{vr}(:,j) - i \times \mathbf{vr}(:,j+1)$.

6: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobvl**, 2: **jobvr**, 3: **n**, 4: **a**, 5: **lda**, 6: **wr**, 7: **wi**, 8: **vl**, 9: **ldvl**, 10: **vr**, 11: **ldvr**, 12: **work**, 13: **lwork**, 14: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, the QR algorithm failed to compute all the eigenvalues, and no eigenvectors have been computed; elements $i+1 : \mathbf{n}$ of **wr** and **wi** contain eigenvalues which have converged.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A+E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.8 of Anderson *et al.* 1999 for further details.

8 Further Comments

Each eigenvector is normalized to have Euclidean norm equal to unity and the element of largest absolute value real and positive.

The total number of floating-point operations is proportional to n^3.

The complex analogue of this function is f08nn.

9 Example

```
jobvl = 'No left vectors';
jobvr = 'Vectors (right)';
a = [0, 0.45, -0.14, -0.17;
     0, 0.07000000000000001, -0.54, 0.35;
     0, -0.33, -0.03, 0.17;
     0.25, -0.32, -0.13, 0.11];
[aOut, wr, wi, vl, vr, info] = f08na(jobvl, jobvr, a)
```

```
aOut =
   -0.1280   -0.1651   -0.3690    0.2446
    0.6753   -0.1280   -0.1836   -0.1114
         0         0   -0.0509    0.1800
         0         0         0    0.4569
wr =
   -0.1280
   -0.1280
   -0.0509
    0.4569
wi =
    0.3339
   -0.3339
         0
         0
vl =
     0
vr =
   -0.2708    0.3271   -0.3282   -0.6887
    0.1909   -0.3732   -0.3848   -0.6091
    0.2941   -0.2549   -0.5282    0.3853
    0.7018         0   -0.6820   -0.0788
info =
           0
```